

Analysis of Energy Thickness of Flow over Various Stepped Chute Geometries

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ABSTRACT

The research has analyzed the significance of energy thickness of flow over stepped chute for the purpose of providing firsthand information for the overall design of the length of stilling basin at the toe of the stepped chute. The values of the energy thickness, Reynolds and Froude Numbers were determined. The results showed that Energy thickness increases with increase in both the Reynolds and the Froude numbers, respectively. A model was developed which relates the energy thickness with the Reynolds and Froude numbers; the values of these terms were analyzed by using multiple regression model in order to determine the values of the coefficients in the model. The analysis showed that deviation from the mean for all the terms range from 5.32 to 9.19%, the error was 0.53% and R^2 was 99%. The values of the energy model were compared with the actual values; which showed a good agreement between them and the error ranges from 0.19 to 2.8%. The research reveals that the energy thickness is about 2% of the value of the boundary layer thickness.

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Introduction

Energy thickness has been defined as the distance measured perpendicular to the boundary of the solid body by which the boundary should be displaced to compensate for the reduction of kinetic energy of the flowing fluid on account of boundary layer formation. Flow over spillway moves with high velocity that may undermine the structure by gradually causing scouring of the channel bed at the toe of the hydraulic structure. The introduction of stepped spillway reduces this high kinetic energy which eventually minimized the cost of the energy dissipator to be installed below the toe of the spillway. The stationary spillway with high velocity of flow over its chute causes shear stress and as a result there exist boundary layer formation from the leading edge of the bed to a point where the boundary layer thickness is equivalent to the water surface as shown in figure 1. As a result there is reduction in the value of this high kinetic energy, which is a useful parameter require to ascertain the length of a hydraulic jump to have caused to occur downstream of the spillway. The determination of energy thickness will help to compensate for the reduction of this high kinetic energy, which in turn will help in the design of the length of stilling basin at the toe of the spillway. Only few works are at the researchers' disposal and such studies were focusing on only the boundary layer thickness except that of [1], where they used energy thickness in prediction of throat heat transfer in rocket nozzles. The energy thickness is one of the elements of boundary layer thickness, δ shown in figure 1. Some

of the researchers that worked on boundary layer thickness include [2 – 14]. The development of empirical equation to be used to determine the energy thickness of flow over any of the chutes, in this research, will serve as a design guide for such structures.

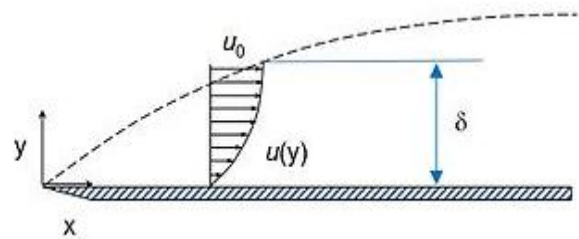


Figure 1. Boundary layer thickness

Materials and Methods

The physical models were used by [15] at the hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University Zaria. The models are designated as stepped chute model (SCM) as shown in Table 1 and Figure 2 respectively. Z =model height, S_h = step height= h_p , height for plain step; the heights for end-sill and inclined steps are respectively, h_p+h_e and h_p+h_i , where h_e and h_i are heights of end-sill and inclined steps in that order. K_s (step roughness height) = $S_h \cos\theta$ for plain and inclined steps, because

of the thickness of the end-sill, the angle used to determine K_s were 46.9° for SCM-3 and 53.1° for SCM-4 taken from Munta (2014). L (Length of the chute) = Z (model height)/ $\sin\theta$ and θ =chute angle.

Table 1. Geometry of the developed stepped Spillway Models

Model No.	Chute shape	Z (cm)	S _h (cm)	K _s (cm)	L (cm)	θ (deg.)
SCM-1	Plain	104	4.0	3.12	166.3	38.7
SCM-2	Plain	104	4.0	2.83	147	45
SCM-3	End-sill	104	8.0	5.47	166.3	38.7
SCM-4	End-sill	104	8.0	4.80	147	45
SCM-5	Inclined	104	8.0	6.24	166.3	38.7
SCM-6	Inclined	104	8.0	5.66	147	45

Source: [15]

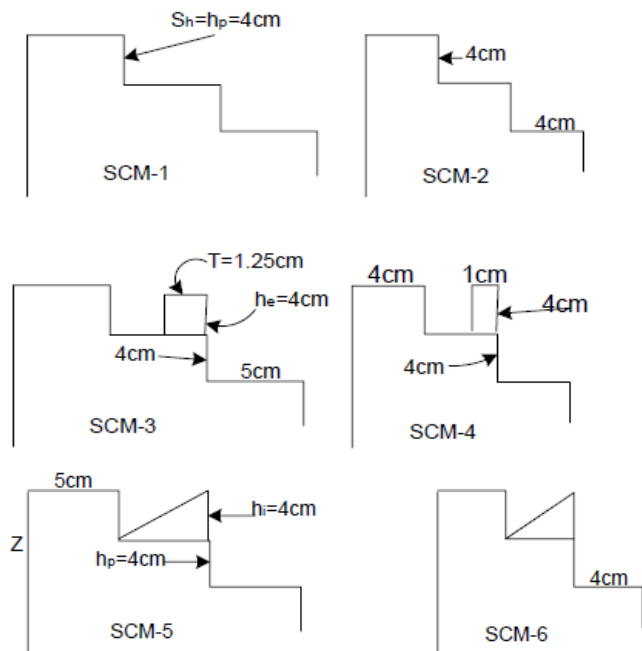


Figure 2. Various stepped chute dimensions [15]

Energy thickness e_T depends on acceleration due to gravity g , free stream velocity, U_0 , boundary layer length L_b , kinematic viscosity ν , boundary layer thickness δ , step roughness height K_s .

$$e_T = f_1(g, U_0, L_b, \nu, K_s) \quad (1a)$$

$$f_2(e_T, g, U_0, L_b, \nu, K_s) = 0 \quad (1b)$$

Selecting gK_s as repeating variables

$$f_2 \left(e_T / K_s, U_0 / \sqrt{gK_s}, L_b / K_s, \nu / \sqrt{gK_s^3} \right) = 0 \quad (1c)$$

$$e_T = \phi \left(U_0 / \sqrt{gK_s}, U_0 L_b / \nu \right) \quad (1d)$$

$$e_T / K_s = C(Fr, Re) \quad (1e)$$

$$e_T = \alpha Fr^\beta Re^\gamma \quad (2)$$

Equation 2 is the energy thickness model under consideration. The coefficients α , β and γ are determined by multiple linear regression model. K_s is assumed to be built up into the values of Fr and Re , hence

$$Fr = U_0 L_b / \sqrt{gK_s} = \text{Froude Number and}$$

$$Re = U_0 L_b / \nu = \text{Reynolds Number.}$$

All other variables in Eq. 1d were previously defined. Kinematic viscosity of water at the temperature of 26°C is given as $8.75\text{E-}07$. The values of the Stream flow velocity, boundary layer thickness and boundary layer length were abstracted from the experiment conducted by Munta (2014) as indicated in Table 2. The value of energy thickness can be determined by the velocity distribution in the turbulent boundary layer given by log law or Prandtl's one seventh power law as shown in Eqs. 3 and 4.

$$\frac{u}{U_m} = \left(\frac{z}{\delta} \right)^{1/7} \quad (3)$$

Hence,

$$e_T = \int_0^\delta \left[\frac{u}{u_0} \left(1 - \frac{u^2}{u_0^2} \right) \right] dz$$

$$= \int_0^\delta \left(\frac{z}{\delta} \right)^{1/7} \left[1 - \left(\frac{z}{\delta} \right)^{2/7} \right] dz$$

$$= \int_0^\delta \left[\left(\frac{z}{\delta} \right)^{1/7} - \left(\frac{z}{\delta} \right)^{3/7} \right] dz$$

$$= \left[\frac{7}{8} \times \frac{z^{8/7}}{\delta^{1/7}} - \frac{7}{10} \times \frac{z^{10/7}}{\delta^{3/7}} \right]_0^\delta$$

$$= \left[\frac{7}{8} \delta - \frac{7}{10} \delta \right] = \frac{7}{40} \delta \quad (4)$$

Table 2: Data of stream flow velocity, boundary layer thickness and length for various models

SCM-1				SCM-2				SCM-3			
q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)	q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)	q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)
26.1	279	1.03	51.1	26.1	292	0.99	49.3	26.1	272	1.06	48.2
31.5	301	1.16	58.5	31.5	314	1.11	56.4	31.5	292	1.20	55.4
35.6	313	1.26	63.8	35.6	328	1.20	61.7	35.6	305	1.30	60.8
43.4	336	1.43	73.8	43.4	253	1.36	71.2	43.4	328	1.47	70.1
SCM-4				SCM-5				SCM-6			
q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)	q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)	q (L/s)	Uo (cm/s)	δ (cm)	Lb (cm)
26.1	285	1.02	46.9	26.1	270	1.07	47.6	26.1	283	1.03	46.2
31.5	306	1.15	53.8	31.5	290	1.21	54.9	31.5	303	1.16	53.0
35.6	319	1.24	58.7	35.6	303	1.31	60.1	35.6	317	1.25	57.8
43.4	343	1.40	67.9	43.4	326	1.48	69.3	43.4	341	1.42	67.0

Results and Discussion

e_T , Fr and Re were determined and displayed in figures 3a to 4c. These values were imported into the Multiple Regression Model in order to determine the values of the coefficients in Eq. 3 as indicated in Eq. 5. The results in figures 3a to 4c show that e_T increases as both the Fr and Re increase. The analysis of the terms in Eq. 3 shows that deviation from the mean for all the terms range from 5.32 to 9.19%. This shows the compatibility of the values of the terms in the equation. The multiple regression model summary gave an error of 0.53% and R^2 of 99%.

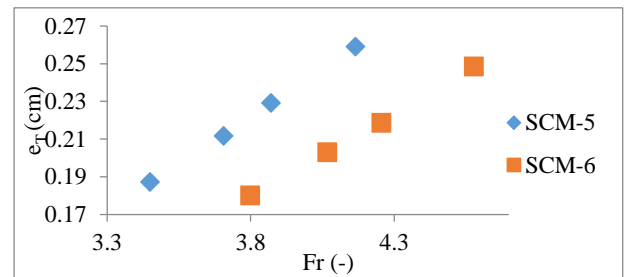


Figure 3c. e_T versus Fr for inclined steps

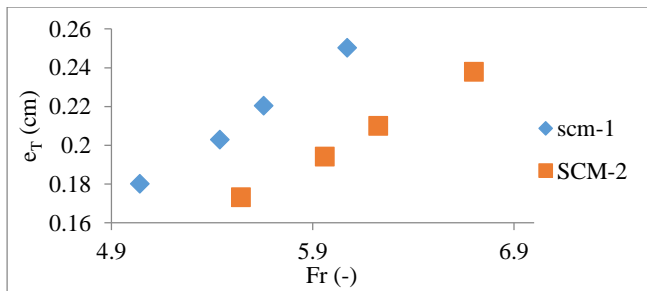


Figure 3a. e_T versus Fr for plain steps

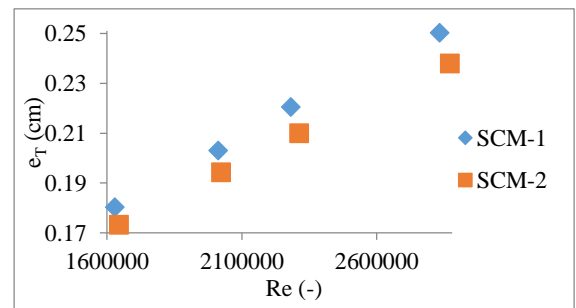


Figure 4a. e_T versus Re for plain steps

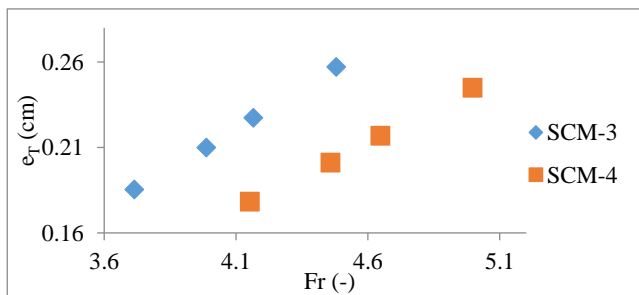


Figure 3b. e_T versus Fr for end-sill

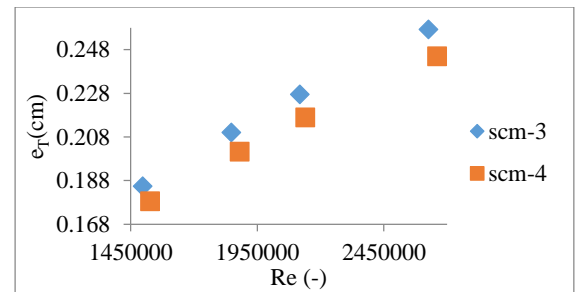


Figure 4b. e_T versus Re for end-sill

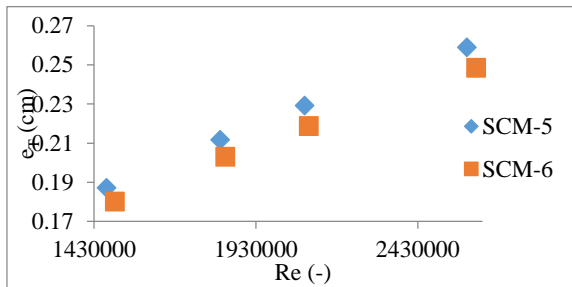


Figure 4c. e_T versus Re for inclined steps

The energy thickness of Eq. 5 was verified by comparing the actual values with the values in the equation as shown in figures 5a to 7b. It shows that there is a reasonable agreement between the actual values of the energy thickness and that obtained from its equation, with error ranging from 2.8% to 0.19%.

$$e_T = \frac{1.694 \times 10^{-5} Re^{0.691}}{Fr^{0.302}} \quad (5)$$

Conclusion

The energy thickness model developed in this study may be used for the prediction of energy thickness of flow over any of the stepped chute consideration. The equation revealed that the contribution of the value of Froude No. term to energy thickness value was 0.00036%, while that of Reynolds No. term was 99.99%. The conclusions drawn in this study are that the gravity force is insignificant within the vicinity of the boundary layer hence it is completely dominated by the viscous force and the value of energy thickness is about 2% of the value of the boundary layer thickness.

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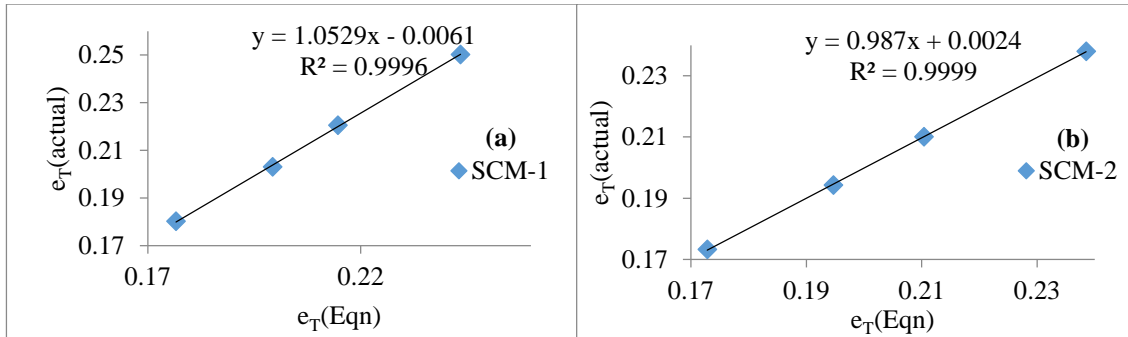


Figure 5 (a, b). Comparison of actual value of energy thickness and that from equation for plain steps

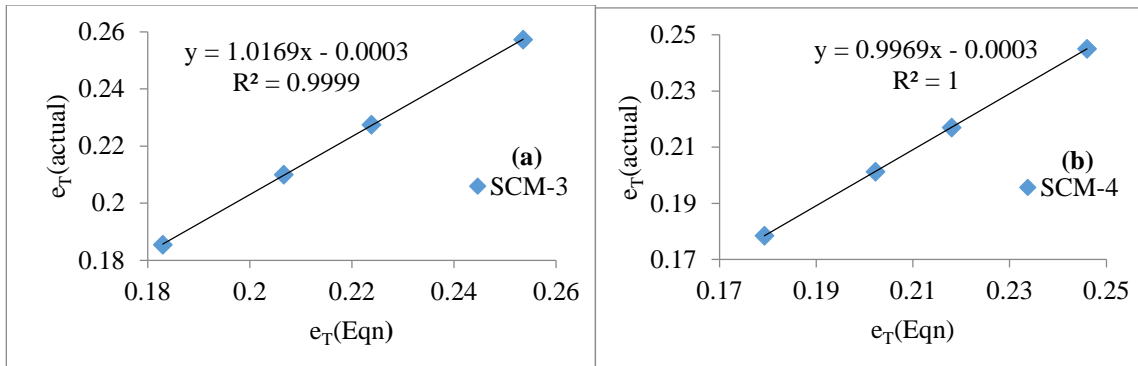


Figure 6 (a, b): Comparison of actual value of energy thickness and that from equation for end-sill

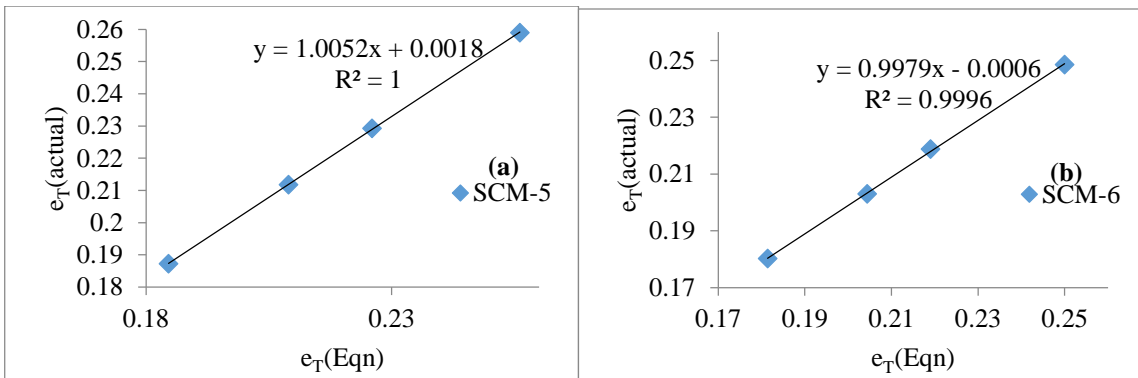


Figure 7 (a, b): Comparison of actual value of energy thickness and that from equation for inclined steps