

# Mathematical Modelling of Coefficient of Discharge for Rectangular Weirs with Various Widths and Heights

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## ABSTRACT

Four physical models of rectangular weir, of variable length and height, were used in the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University, Zaria, to run the discharge-head experiment. The Coefficient of discharge for every stage of the run of each model was determined. Coefficient of discharge model was also developed, where it depended on Reynolds number, flow depth with respect to the length and height of the weirs correspondingly. The values of the terms in the equation were analyzed using SBM, SPSS 21 to determine the unknown in the developed coefficients of discharge model. The reliability of the coefficient of discharge model was 99.8% and error of 0.08%. The results have shown that there was a good agreement between the actual coefficient of the discharge and from equation, with error ranging from 0 to 0.43%. The result also revealed that the Reynolds number contributed about 10.4% to the value of the coefficient of discharge model, indicating that the viscous force has some influence on the value of coefficient of discharge. Lastly, the empirical equation in this study could be employed to predict coefficient of discharge of the hydraulic physical models under consideration.

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## Introduction

A rectangular weir is one of the raised structures placed across a flowing channel for the purpose of regulating, measuring or diverting flow in hydraulics, irrigation and environmental projects. For flow measurement, the sill or height of the weir continues to obstruct the flow creating prevailing depth until it overflows the crest of the weir; also the width of the weir influences the flow and finally the layers of the fluid cause shear stress between them. Many relationships were developed to relate coefficient of discharge ( $C_d$ ) with some geometrical, flow and fluid properties, among which is [1] who presented an expression as shown in Eq. 1.

$$C_d = a + b \left( \frac{h}{w} \right) \quad (1)$$

where,  $h$  is the head over the crest,  $w$  is the weir height and  $a$  and  $b$  are coefficients to develop a discharge equation for the sharp-crested curved weir. Reference [2] determined the relationship shown in Eq. 2.

$$C_d = \frac{H_t}{P} \quad (2)$$

It is for quarter round crests trapezoidal labyrinth weirs.

Reference [3] presented an equation given as shown in Eq. 3. This is for normal and oblique weirs with semi-circular crest configuration of varying crest radius ( $R$ ) of 5 cm, 7.5 cm and 10 cm, varying weir height  $P$  of 20 cm, 25 cm, 30 cm, and varying oblique angle  $\alpha$  of 30°, 45°, 60°, and 90°.

$$C_d = \frac{0.674}{(H/P)^{0.15} (R/P)^{0.54} (\sin \alpha)^{0.86}} \quad (3)$$

Also, [4] presented a relationship as shown in Eq. 4.

$$C_d = \phi \left( \frac{H_w}{R}, D, \beta \right) \quad (4)$$

This is the experimental study that examines the influence of geometrical characteristics of the oblique cylindrical weir on coefficient of discharge.

Reference [5], through their laboratory study, investigated the discharging capacity of a sharp-crested weir of curved plan form under free flow condition in a rectangular channel and developed an equation of coefficient of discharge as shown in Eq. 5.

$$C_d = \left( 0.5 + 0.33 \frac{h}{w} + \frac{h}{L} \right)^{0.06} \quad (5)$$

Reference [6] developed an equation for three models of broad crested weir of varying lengths of 20, 25 and 30 cm

respectively but with constant height of 10cm and width of 7.6 cm as shown in Eq. 6.

$$C_d = 0.714 + 0.86 \frac{H}{L} - 49.606 \frac{H}{K_s} \quad (6)$$

where  $K_s$  = roughness height.

Reference [7] presented an equation on semicircular crested oblique weirs as shown in Eq. 7.

$$C_d = K \left( \frac{h}{P} \right)^{\beta_1} \left( \frac{R}{P} \right)^{\beta_2} (\sin \alpha)^{\beta_3} \quad (7)$$

where,  $P$  = constant crest height=20cm,  $R$  = variable crest radius (5, 7.5, 10 cm),  $\alpha$  = variable angle of inclination (15, 30, 60 90°).

In all the stated models studied, only [6] wanted to include Reynolds and Weber numbers respectively, in their equation but declined, that they had no influence on the value of coefficient of discharge of such structures. The focus of this study is to take into account the influence of Reynolds number in the equation that will be developed in this research.

### Materials and Methods

The discharge measurements were excerpts from works of [8, 9], respectively, carried out at the Hydraulics Laboratory of the Department of Water Resources and Environmental Engineering, Ahmadu Bello University, Zaria. Four Rectangular Weir Models (RCM) were built of wood and fixed to the outlet of a flume, in accordance with the specification that  $Z/H \geq 1.33$  [10], in order to neglect the effect of velocity of approach. Details are shown in figure 1 and Table 1, respectively.

The discharge measurement over a rectangular weir was done by the volumetric method; where the flow head above the crest of the weir for every discharge was measured by a point gauge and was collected inside a container of a known volume after a particular time period. The procedures were repeated for several values of discharges. Hence the actual discharge measured can be determined as shown in Eq. 8.

$$Q_{act.} = V / t \quad (8)$$

where  $Q$  is the actual discharge in l/s, the volume  $V$ , is in litres and the time,  $t$  is in seconds.

The theoretical discharge is given as shown in Eq. 9.

$$Q = \frac{2}{3} C_d \sqrt{2g} LH^{1.5} \quad (9)$$

Equation 9 is of the form

$$Q_{act.} = kH^n \quad (10a)$$

where,  $Q_{act.}$  = the actual discharge,  $H$  is the head of water above the crest,  $k$  and  $n$  are coefficients related to the discharge.

Taking the logarithm of the terms in Equation (10a), it becomes:

$$\text{Log}Q_{act.} = \text{Log}K + n\text{Log}H \quad (10b)$$

$k$  and  $n$  in Eq. 10b are determined by plotting  $\text{Log} Q_{act.}$  against  $\text{Log} H$  on excel spreadsheet 2010. Comparing Eqs. 9 and 10a,

$$C_d = \frac{Q_{ac}}{Q} = \frac{K}{2/3 \times L \times \sqrt{2g}} \quad (10c)$$

Eq. 10c holds when  $n$ , in Eq. 10a is assumed to be equals to 1.5 with permissible error of 0.5%.

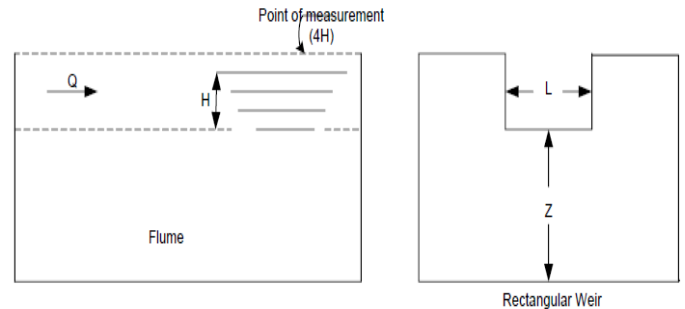


Figure 1: Typical flow experiment over a rectangular weir

Table 1. Geometrical features of various Rectangular Weirs

Source	Munta (2014)	Munta (2014)	Munta (2010)	Munta (2010)
Model Number	RCM 1	RCM 2	RCM 3	RCM 4
Height, Z (cm)	12	14	16	24
Width, L (cm)	12	20	25	30
Temp., T (°C)	26	24	23	22
Kinematic vis., $\nu$ (m <sup>2</sup> /s)	8.75E-07	9.15E-07	9.36E-07	9.58E-07

The discharge  $Q$  over a rectangular weir is depends on the head ( $H$ ) of water over the crest, height of the weir ( $Z$ ), its width ( $L$ ), acceleration due to gravity ( $g$ ) and kinematic viscosity ( $\nu$ ) of the flow. Mathematically,

$$Q = (H, Z, L, g, \nu) \quad (11)$$

$$\text{Or } f_1 = (Q, H, Z, L, g, \nu) = 0 \quad (12a)$$

Considering,  $gH$  as repeating variables, Eq. 12a may be re-written as shown in Eq. 12b.

$$f_2 = (\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad (12b)$$

$$\left. \begin{aligned} \pi_1 &= g^\alpha H^\beta Q \\ \pi_2 &= g^\alpha H^\beta Z \\ \pi_3 &= g^\alpha H^\beta L \\ \pi_4 &= g^\alpha H^\beta \nu \end{aligned} \right\} \quad (12c)$$

$\alpha$  and  $\beta$  in Eq. 12c are determined by the principle of dimensional homogeneity. Substituting the values of  $\pi_1, \pi_2, \pi_3$  and  $\pi_4$  in Eq.

(12b), the functional relationship becomes as show in Eq. 12d or 12e.

$$f_2 = \left( \frac{Q}{g^{1/2} H^{5/2}}, \frac{L}{H}, \frac{Z}{H}, \frac{v}{g^{1/2} H^{3/2}} \right) \quad (12d)$$

$$\frac{Q}{g^{1/2} H^{5/2}} = \phi \left( \frac{v}{g^{1/2} H^{3/2}}, \frac{L}{H}, \frac{Z}{H} \right) \quad (12e)$$

$\frac{Q}{AV}$  is also dimensionless and for a rectangular weir,  $A \propto LH, V \propto g^{1/2} H^{1/2}$ . Hence, by transforming the left hand term of Eq. 12e, it becomes

$$\frac{Q}{Lg^{1/2} H^{3/2}} = \phi \left( \frac{v}{g^{1/2} H^{3/2}}, \frac{L}{H}, \frac{Z}{H} \right) \quad (12f)$$

Eq. 12f can also be re-written as

$$\frac{Q}{g^{1/2} LH^{3/2}} = \phi \left( Re, \frac{H}{L}, \frac{H}{Z} \right) \quad (12g)$$

Also, Eq. 12g is of the form

$$Q = \frac{2}{3} C_d \sqrt{2g} LH^{3/2} \quad (12h)$$

Comparing Eqs. (12g) and (12h), Eq. 12i is obtained

$$C_d = \phi \left( Re, \frac{H}{L}, \frac{H}{Z} \right) \quad (12i)$$

Furthermore, Eq. 12i can be re-written in the form shown in Eq. 13.

$$y = k(x_1^a, x_2^b, x_3^c) \quad (13)$$

Eq. (13) is considered to be the coefficient of discharge model, where,  $y = C_d, x_1 = Re = \frac{H\sqrt{gH}}{v}$

(Reynolds number),  $x_2 = H/L =$  ratio of head of water to the crest's length,  $x_3 = H/Z =$  ratio of water head to height of the weir,  $k, a, b, c$  are coefficients which are determine by using IBM SPSS 21.

Results of coefficient of discharge as a function of the Reynolds number, ratio of the head to width and ratio of head to the height are shown in figures 3 to 5 respectively. The results reveal that  $C_d$  decreases with increase in  $Re$  for the different models. Also,  $C_d$  increases as  $H/L$  decreases. Finally  $C_d$  decreases

with increase in  $H/P$ . The logarithm of the results in figures 3 to 5 were analyzed using IBM SPSS MODEL 21 to determine the unknown in Eq. 13 which is displayed in Eqn. 14.

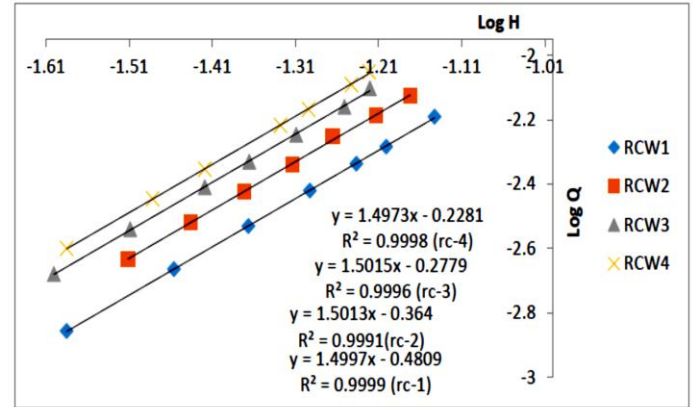


Figure 2. Discharge - Head relationship

Table 2: Values of co-efficient of discharges for the various Rectangular Weirs

No. of Runs	RCM 1	RCM 2	RCM 3	RCM 4
1	0.742	0.720	0.703	0.667
2	0.743	0.721	0.705	0.669
3	0.745	0.723	0.707	0.670
4	0.746	0.725	0.708	0.672
5	0.747	0.727	0.709	0.673
6	0.748	0.728	0.710	0.675
7	0.749	0.731	0.713	0.677

$$y = 2.50 \frac{x_2^{0.058} x_3^{0.111}}{x_1^{0.123}} \quad (14)$$

The reliability  $R^2$ , of the coefficient of the discharge model (Eq. 14) is 0.998 with an error of 0.077% as revealed by the regression model. Furthermore, the actual  $C_d$  from measurements were compared with the ones from Eqn. (14), Table 4, and it reveals that there is a good agreement between the measured values and those from equation, in addition, the errors between the values of the  $C_d$  ranges from 0 to 0.43%.

## Conclusion

A coefficient of discharge model was established in this study which could be used to predict the coefficient of discharge of the models under consideration. The study has shown that the  $Re, H/L$  and  $H/P$  influences the value of  $C_d$  in the order of 10.4%, 39.4% and 50.2%, which signifies that  $Re$  has some effects to the contribution of the value of  $C_d$ , while  $H/Z$  contributed significantly.

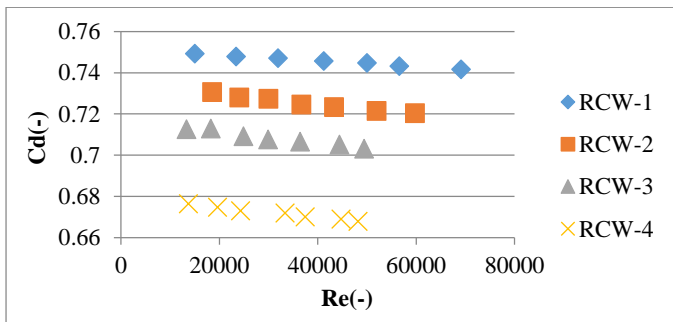


Figure 3:  $C_d$  as a function of  $Re$

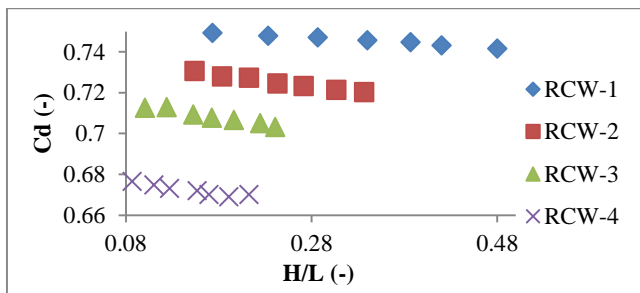


Figure 4:  $C_d$  as a function of  $H/L$

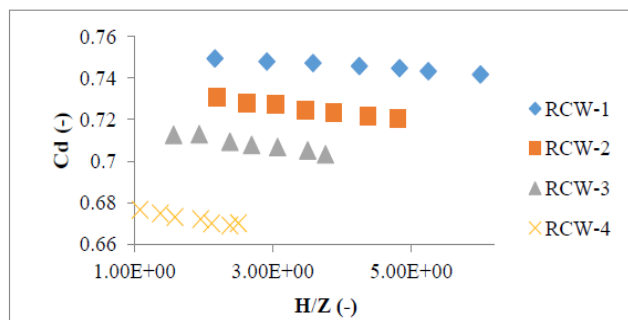


Figure 5:  $C_d$  as a function of  $H/Z$

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Table 4. Model Validation

S/N	RCM 1			RCM 2			RCM 3			RCM 4		
	Act.	Theo.	% error	Act.	Theo.	% error	Act.	Theo.	% error	Act.	Theo.	% error
1	0.742	0.741	0.12	0.720	0.721	0.07	0.703	0.704	0	0.668	0.668	0
2	0.743	0.742	0.12	0.721	0.722	0.06	0.705	0.705	0	0.669	0.669	0
3	0.745	0.743	0.20	0.723	0.723	0	0.707	0.707	0	0.670	0.670	0
4	0.746	0.745	0.12	0.725	0.724	0.01	0.708	0.708	0	0.672	0.671	0.21
5	0.747	0.747	0	0.727	0.726	0.19	0.709	0.709	0	0.673	0.673	0
6	0.748	0.749	0.16	0.728	0.728	0	0.712	0.712	0	0.675	0.674	0.72
7	0.749	0.752	0.43	0.731	0.730	0.13	0.713	0.714	0.17	0.677	0.677	0